

Theoretically motivated dark electromagnetism as the origin of relativistic MOND

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ABSTRACT

The present paper is a modest attempt to initiate the research program outlined in this abstract. We propose that general relativity and relativistic MOND (RelMOND) are analogues of the broken electroweak symmetry. That is, $SU(2)_R \times U(1)_{YDEM} \rightarrow U(1)_{DEM}$ (DEM stands for dark electromagnetism), and GR is assumed to arise from the broken $SU(2)_R$ symmetry, and is analogous to the weak force. RelMOND is identified with dark electromagnetism $U(1)_{DEM}$, which is the remaining unbroken symmetry after spontaneous symmetry breaking of the darkelectro-grav sector $SU(2)_R \times U(1)_{YDEM}$. This sector, as well as the electroweak sector, arise from the breaking of an $E_8 \times E_8$ symmetry, in a recently proposed model of unification of the standard model with pre-gravitation, this latter being an $SU(2)_R$ gauge theory. The source charge for the dark electromagnetic force is square-root of mass, motivated by the experimental fact that the square-roots of the masses of the electron, up quark, and down quark, are in the ratio 1:2:3, which is a flip of their electric charge ratios 3:2:1. The introduction of the dark electromagnetic force helps understand the weird mass ratios of the second and third generation of charged fermions. We also note that in the deep MOND regime, acceleration is proportional to square-root of mass, which motivates us to propose the relativistic $U(1)_{DEM}$ gauge symmetry as the origin of MOND. We explain why the dark electromagnetic force falls inversely with distance, as in MOND, and not as the inverse square of distance. We conclude that dark electromagnetism is a good mimicker of cold dark matter, and the two are essentially indistinguishable in those cosmological situations where CDM is successful in explaining observations, such as CMB anisotropies, and gravitational lensing.

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I. INTRODUCTION

A theory of unification of the fundamental forces has recently been proposed [1], starting from the foundational requirement that there should exist a reformulation of quantum field theory, which does not depend on classical time [2]. This theory is based on an $E_8 \times E_8$

symmetry group, in which each of the two E_8 groups is assumed to branch as follows, as a result of a spontaneous symmetry breaking which is identified with the electroweak symmetry breaking:

$$E_8 \longrightarrow SU(3) \times E_6 \longrightarrow SU(3) \times SU(3) \times SU(3) \times SU(3) \longrightarrow \quad (1)$$

$$SU(3) \times SU(3) \times SU(3) \times SU(2) \times U(1) \quad (2)$$

Leaving the two E_6 aside for a moment, the $SU(3) \times SU(3)$ pair arising from the $E_8 \times E_8$ branching is mapped to an $(8+8=16)$ dimensional split bioctonionic space from which our 4D spacetime as well as the internal symmetry space for the standard model forces (and two newly predicted forces) are assumed to emerge.

The three $SU(3)$ s arising from branching of each of the two E_6 , with the right-most $SU(3)$ in each set branching as $SU(2) \times U(1)$, are interpreted as follows. In the first E_6 , the first $SU(3)$ is $SU(3)_{genL}$ and describes three generations of left-handed standard model fermions (eight per generation, along with their anti-particles). The second $SU(3)$ is associated with $SU(3)_{color}$ of QCD. The branched third $SU(3) \rightarrow SU(2)_L \times U(1)_Y$ describes the electroweak symmetry of the standard model, broken to $U(1)_{em}$.

In the second of the two E_6 , the first $SU(3)$ is $SU(3)_{genR}$ and describes three generations of standard model right-handed fermions including three types of sterile neutrinos (eight fermions per generation, along with their anti-particles). The second $SU(3)$ is identified with a newly predicted but yet to be discovered new (likely short-range) ‘sixth force’ named $SU(3)_{grav}$. The third $SU(3) \rightarrow SU(2)_R \times U(1)_{YDEM}$ describes what we call the darkelectrograv sector which breaks to the newly predicted ‘fifth force’ $U(1)_{DEM}$ which we name dark electromagnetism, which we propose to be the relativistic MOND theory (a gauge theory) whose non-relativistic limit is Milgrom’s MOND [3]. The broken $SU(2)_R$ symmetry is proposed to give rise to classical gravitation described by the general theory of relativity (GR). At low accelerations, the fifth force of dark electromagnetism (DEM) dominates over GR, whereas at high accelerations GR dominates over DEM, with the transition coming at the critical MOND acceleration $a_M \sim a_0/6 \approx cH_0/6$, where a_0 is the cosmological acceleration of the current accelerating universe. We reiterate that standard general relativity is assumed to emerge from the broken $SU(2)_R$ symmetry, whereas $SU(3)_{grav}$ is a newly predicted unbroken symmetry (likely short range and extremely weak, and in which the charged leptons

and down family of quarks take part).

The particle content of this unification proposal has been described in detail in Kaushik et al. [1]. All the $248+248=496$ degrees of freedom of $E_8 \times E_8$ are accounted for. The only fermions in the theory are three generations of standard model chiral fermions. Apart from the 12 standard model gauge bosons, there are 12 newly predicted spin one gauge bosons associated with the $SU(3)_{grav} \times SU(2)_R \times U(1)_{DEM}$ sector. Eight of these are so-called gravi-gluons associated with the (likely to be short range, and ultra-weak compared to QCD) $SU(3)_{grav}$. The gauge boson associated with $U(1)_{DEM}$ is named the dark photon, and is massless and has zero electric charge. Of the three bosons associated with the broken $SU(2)_R$ symmetry, two have zero electric charge but are as massive as Planck mass, and hence mediate Planck length range interaction: these are analogs of the W^+ and W^- bosons of the weak force. The third is massless and has an insignificantly tiny electric charge (scaled down enormously due to cosmological inflation, in comparison to charge of the electron) which can be set to zero for all practical purposes. This boson is the analog of the Z^0 of the weak force. Pre-gravitation $SU(2)_R$ symmetry is mediated by spin-one gauge bosons, with gravitation as described by the metric tensor in the general theory of relativity emerging only in the classical limit. In our approach, classical GR is not to be quantised, which is why we do not have a fundamental, non-composite, spin 2 graviton in the theory. This does not contradict the fact that classical GR admits the experimentally confirmed quadrupolar gravitational waves. The apparent spin 2 nature of gravitation is emergent only in the classical limit. The underlying theory from which spacetime and GR emerge in the classical limit is a pre-quantum, pre-spacetime theory. Gravitation, and quantum theory, are emergent phenomena.

There are two Higgs doublets in this theory, one being the standard model Higgs which gives mass to left-chiral fermions upon spontaneous breaking of the electroweak symmetry. The second, newly predicted Higgs boson gives electric charge to the right chiral fermions, upon breaking of the darkelectro-grav symmetry, which coincides with the electroweak symmetry breaking. Unlike in the standard model, both the Higgs are now predicted to be composite, being composed of the very fermions to which they give mass and electric charge. Of the 496 degrees of freedom in the theory, 32 are with the bosons (after including 4 each for the two Higgs). 32 degrees of freedom are with internal generation space and pre-spacetime (16 each), and 144 d.o.f. are with the fermions. The remaining 288 d.o.f. go into making two composite Higgs, 144 per Higgs. It is noteworthy that each Higgs has as many composite

d.o.f. as the total number of d.o.f. in the fermions. The bosonic content of the theory is confirmed also by examining the Lagrangian of the theory, as has been done in Raj and Singh [4].

The source charge associated with $U(1)_{em}$ is of course the electric charge, and in the algebraic approach to unification, it can be shown to be quantised, as was done for instance by Furey [5]. Electric charge is defined as the number operator constructed from generators of the Clifford algebra $Cl(6)$, which algebra is in turn generated by octonionic chains acting on octonions. It is shown that electric charge can only take the values $(0, 1/3, 2/3, 1)$. Furthermore, the spinorial states associated with these charge values exhibit the following symmetry under the group $SU(3)$ (which is a maximal subgroup of the smallest exceptional group G_2 , the automorphism group of the octonions). The states with charges 0 and 1 are shown to be singlets of $SU(3)$, states with charge $1/3$ are anti-triplets, and states with charge $2/3$ are triplets. This enables the interpretation that the state with charge $(0, 1/3, 2/3, 1)$ are respectively the (left-handed) neutrino, anti-down quark, up quark, and positron. The $SU(3)$ is hence identified with $SU(3)_{color}$ of QCD. Anti-particle states are obtained by complex conjugation of particle states and are shown to have opposite sign of electric charge, as anticipated. Note that these fermions are left-chiral particles, and their corresponding antiparticles are right chiral. Furthermore, this quantisation of electric charge holds for every one of the three fermion generations. The Clifford algebra construction applies equally well to the second and to the third generation.

Consider next the symmetry $SU(3)_{grav} \times U(1)_{DEM}$ associated with the right handed sector, with these two being the two new forces [1]. Now, the source charge associated with the $U(1)_{DEM}$ symmetry is square-root of mass $\pm\sqrt{m}$, not electric charge. The motivation for proposing this interpretation (for the number operator made from the Clifford algebra $Cl(6)$ generators which define the right-chiral fermions) comes from the following remarkable fact [6]. The square-roots of the masses of the electron, up quark and down quark are in the ratio 1:2:3, which is a flip of their electric charge ratios 3:2:1. We treat electric charge and square root of mass on the same footing. Square root of mass also takes two signs: $+\sqrt{m}$ and $-\sqrt{m}$. The positive sign is for matter, and negative sign is for anti-matter: like signs attract under dark electromagnetic force, and unlike signs repel. Note that mass m , being the square of $\pm\sqrt{m}$, is necessarily positive. Three new colors for $SU(3)_{grav}$ are introduced: the right-handed neutrino and the down quark are singlets of these new colors, and have

\sqrt{m} value 0 and 1 respectively. The electron is an anti-triplet of $SU(3)_{grav}$ with \sqrt{m} value $1/3$, and the up quark is a triplet of $SU(3)_{grav}$ with \sqrt{m} value $2/3$. Their anti-particles have corresponding square-root mass values $-\sqrt{m}$. This mass quantisation is derived from first principles, just as for electric charge quantisation, and holds for every one of the three generations, again just like for the electric charge. Note that our proposal also gives a dynamical definition for matter / anti-matter: matter has positive sign of square root mass ($+\sqrt{m}$) and anti-matter has negative sign of square-root mass ($-\sqrt{m}$). Mass m is of course positive for matter as well as anti-matter, it being obtained from squaring of $\pm\sqrt{m}$.

Why then do the second and third fermion generations have such strange mass ratios, as observed in experiments [7]? The answer is that even when we do experiments to measure particle masses, the measurements are electromagnetic in nature, and carried out using electric charge eigenstates. These electric charge eigenstates are not eigenstates of (square-root) mass. The exceptional Jordan algebra associated with the three fermion generations (one algebra for the electric charge eigenstates which are left-chiral, and one algebra for the square-root mass eigenstates which are right chiral) can be used to express electric charge eigenstates as superposition of square-root mass eigenstates through the so-called Jordan eigenvalues. The weights of these superpositions reveal the observed mass ratios to a very good accuracy [8], and strongly support the proposal that the source charge associated with the dark electromagnetic force is square root of mass. The fact that the source charge for the MOND acceleration is also square-root of mass encourages us to identify dark electromagnetism with relativistic MOND.

In the very early universe, at the epoch of electroweak symmetry breaking, the enormous repulsive dark electromagnetic force segregated matter ($+\sqrt{m}$) from anti-matter ($-\sqrt{m}$), so that our part of the matter-antimatter symmetric universe is matter dominated [4]. (This scenario bears resemblance to the CPT symmetric universe model proposed by Boyle and Turok [9–11].) As a result, the dark electromagnetic force in our matter-dominated universe is apparently attractive only (even though $U(1)_{em}$ is a vector interaction). Similarly, the emergent gravitational interaction which is the classical limit of the $SU(2)_R$ gauge theory, is attractive only. We predict that the $U(1)_{DEM}$ force between an electron and a positron is repulsive.

Another important aspect of the octonionic theory [2] (i.e. the one based on $E_8 \times E_8$ symmetry) is the ‘square-root of spacetime’. The spinorial states which define the fermions

and satisfy the Dirac equation are constructed from the algebra of the octonions acting on itself. In this sense, a spinor is the square of an octonion, and since spinors are defined on spacetime, this suggests the view that a space which is labeled by using octonions as coordinates is actually square-root of spacetime. However, the absolute square modulus of an octonion should be assigned dimensions of length-squared, not length (as the Lagrangian of the octonionic theory suggests). This compels us to introduce the effective distance $R_{eff}^2 = R_H R$ in the unbroken theory, where R_H is the deSitter horizon. An unbroken symmetry such as $U(1)_{DEM}$ thus ought to have a distance dependence (say in Coulomb's law) as $1/(\sqrt{R_{eff}})^2 \sim 1/R$, and not $1/R^2$. This is a possible explanation for the $1/R$ dependence of the MOND acceleration, which taken together with the source charge for $U(1)_{DEM}$, can explain why the MOND acceleration behaves as \sqrt{M}/R , unlike gravitation which goes as M/R^2 in the Newtonian limit. We can say that gravitation is the square of dark electromagnetism: the source current for DEM is $\sqrt{m}cu^i$ whereas the source current for gravitation is $(\sqrt{m}cu^i)(\sqrt{m}cu^j) = mc^2u^i u^j$ which is nothing but the energy-momentum tensor.

In our proposal for DEM as relativistic MOND, the DEM force mimics Maxwell's electrodynamics, with electric charge replaced by square root of mass, and spatial distance replaced by an effective distance $R_{eff} \equiv \sqrt{RR_H}$ where R_H is the Hubble radius, equivalently the deSitter horizon. The source term, in the non-relativistic limit, is the effective volume density of square-root mass: \sqrt{M}/R_{eff}^3 . The left hand side of the Poisson equation is the Laplacian made using the effective distance function. Such a Poisson equation yields MOND, in the deep MOND regime.

As we explain in Section VI of the paper, our proposal for MOND as dark electromagnetism also explains Verlinde's entropic criterion [12] for MOND. Keeping this connection with Verlinde in mind, in Section III we review Verlinde's proposal for motivating MOND from entropy considerations.

Since MOND has a distance dependence in the acceleration as $1/R$, the associated MOND potential is logarithmic. This is in principle consistent with the source being a surface square-root mass density $\sim \sqrt{M}/R^2$, as if the MOND dynamics were taking place effectively in two spatial dimensions. This is consistent with the logarithmic form for the Green's function of the Laplace equation in 2D, as we recall in Section IV. Note however that this surface density of square-root mass does not have a well-defined limit as $R \rightarrow 0$ (it diverges as

$R^{-1/2}$), whereas the volume density of square root mass defined using the effective distance does have a well-defined limit which is finite. Also, we would not like to modify the structure of the left-hand side of the Poisson equation, and this is consistent with proposing MOND as the non-relativistic limit of the $U(1)_{DEM}$ gauge theory.

Why do we associate the $SU(2)_R$ gauge symmetry with gravitation, as in the general theory of relativity? The following arguments provide a number independent hints in favour of the notion that the group $SU(2)_R$ (arising in the octonionic theory [1, 2, 4]) qualifies to describe a theory of gravity in 4 dimensions.

In the octonionic theory there appears the product group $SU(2)_L \times SU(2)_R$, where one copy is left-handed, the other right-handed. Now $SU(2)_L \times SU(2)_R$ is locally isomorphic to $SO(4)$, the rotation group in 4 Euclidean dimensions. A Wick rotation will transform $SO(4)$ into the Lorentz group $SO(1,3)$. So the Lorentz group in 4 dimensions is locally isomorphic to the product group $SU(2)_L \times SU(2)_R$.

That the left-handed subgroup $SU(2)_L$ accounts for the weak interaction within the standard model has been known for long. Here we claim that the right-handed subgroup $SU(2)_R$ can account for gravity in 4 dimensions.

To see how a graviton could possibly arise in this setting, consider the tensor product $\mathbf{1} \otimes \mathbf{1}$ of 2 copies of the 3-dimensional irrep of $SU(2)$. Now $\mathbf{1} \otimes \mathbf{1} = \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{0}$. The $\mathbf{2}$ representation carries spin 2 and can thus accommodate the graviton. We expect the $\mathbf{2}$ irrep to accommodate the emergent, spin 2 graviton, with the $\mathbf{1}$ being the gravitational analogue of the electroweak W^\pm and Z^0 . The $\mathbf{0}$ irrep might be the standard model Higgs.

Moreover, Fermi's phenomenological theory of weak interactions has a Lagrangian that carries the Fermi constant G_F multiplying the product of 2 currents; the dimension of G_F is $[\text{energy}]^{-2}$. On the other hand, general relativity has a Lagrangian that carries Newton's constant G_N , the coupling constant being actually the inverse $1/G_N$. Incidentally the dimension of G_N is again $[\text{energy}]^{-2}$. However G_N appears downstairs within its Lagrangian, as opposed to G_F which appears upstairs.

That both G_F and G_N are dimensionful makes the corresponding theories nonrenormalisable. Since the two are effective theories (low-energy limits of more fundamental theories), nonrenormalisability is not an issue.

All these hints make one suspect that gravity and the weak force could share a common origin, namely, the group $SU(2)_L \otimes SU(2)_R$ within the octonionic theory. That the prod-

uct of the two coupling constants G_F and $1/G_N$ is *dimensionless*, suggests the intriguing possibility that Fermi's theory and general relativity might be each other's dual under a Z_2 duality transformation exchanging the weak and the strong coupling regimes. This duality is strongly reminiscent of analogous dualities put forward in the literature [13, 14].

Altogether, this allows one to think of $SU(2)_L$ as the dual of the theory governed by $SU(2)_R$. Gravity would then appear as the weak dual of the Fermi theory, the latter being the strong counterpart. Mention should also be made of the several attempts that have been made in the past, on gravi-weak unification [15].

Further evidence for a possible connection between the $SU(2)_R$ gauge symmetry and gravity comes from the work of Ashtekar [16], of Krasnov [17], and of Woit [18, 19]. There is also the attractive fact that $SU(2)_R \times U(1)_{YDEM}$ (i.e. darkelectro-grav) is a renormalisable gauge theory, just as the electroweak theory $SU(2)_L \times U(1)_Y$ is.

The cosmological setting for our proposal of dark electromagnetism is as follows [20]. Subsequent to the big-bang creation event, the universe undergoes an inflation-like expansion. The expansion begins with a Planck-scale acceleration $\sim 10^{53} \text{ cm s}^{-2}$, and the acceleration falls inversely with the expanding scale factor. One input taken from observations is that the universe has $N \sim 10^{80}$ particles and hence a total mass of about 10^{55} g . The inflating epoch undergoes a phase transition when the decreasing acceleration equals the surface gravity of a black hole with the same mass as the mass of the observed universe. This acceleration happens to be of the same order as the critical MOND acceleration $\sim 10^{-8} \text{ cm s}^{-2}$, as also the acceleration of the current universe. Hence there is an inflation of the scale factor by 61 orders of magnitude before the inflation-like phase ends. (Incidentally, this inflation by 61 orders of magnitude brings down the cosmological constant - which has dimensions of inverse squared length - by 122 orders of magnitude, to the same order as its currently observed value). This phase transition is also a quantum-to-classical transition, and because black hole surface gravity is now higher than the inflationary acceleration, classical inhomogeneous structures can begin to form, and classical spacetime obeying the laws of general relativity emerges. This transition is also the electroweak symmetry breaking and the darkelectro-grav symmetry breaking. Near compact objects, GR as emergent from the broken $SU(2)_R$ symmetry dominates; whereas far from compact objects (once the induced acceleration falls below the critical MOND acceleration) the unbroken symmetry $U(1)_{DEM}$ of dark electromagnetism dominates. This latter is the deep MOND regime. Thus, in the

presence of compact objects, the deSitter horizon does not immediately yield to GR; rather the MOND zone mediates between the GR zone and the horizon. It is as if there is a phase transition between the GR zone and the MOND zone (similar to Verlinde’s ideas [12]). This might be explicable via a generalisation of ‘GR as thermodynamics’ to ‘(GR + MOND) as thermodynamics’ of an unbroken symmetry phase transforming to a broken symmetry phase. The GR dominated phase exhibits the broken symmetry phase and is stiff; the MOND phase is that of unbroken symmetry and is elastic - the deep MOND region carries a memory of the unbroken inflation-like phase, and also of the currently accelerating universe.

We note that grand unification (GUTs) models based on E_6 symmetry have been considered by several researchers before [21–23], and the significance of E_6 has been noted repeatedly (it is the only exceptional Lie group which has complex representations). Our proposal, the octonionic theory, is not a GUT. We have an $E_6 \times E_6$ unification of standard model forces with gravitation, and we predict two new forces, $SU(3)_{grav}$ and $U(1)_{DEM}$. The inflation-like expansion resets the scale of quantum gravity from the Planck scale to the scale of electro-weak symmetry breaking, i.e. ~ 1 TeV. This is also the scale of the breaking of the darkelectro-grav symmetry $SU(2)_R \times U(1)_{YDEM}$, when space-time and gravitation emerge from the pre-quantum, pre-spacetime theory. Relativistic MOND $U(1)_{DEM}$ also emerges at this epoch.

The term dark electromagnetism / dark radiation / dark photon, is sometimes used to refer to a hypothetical radiation which mediates interactions between dark matter particles. In our proposal however, this dark radiation mediates a fifth force between ordinary baryonic matter particles (and of course between leptons as well). There is no dark matter in our theory, unless one wishes to refer to the dark photons of DEM as dark matter.

II. A BRIEF REVIEW OF MOND AND RELATIVISTIC MOND

The flattened rotation curves of galaxies are non-Keplerian [24], and it is observed that departure of the rotation curve from Newtonian gravity sets in whenever the observed acceleration falls below the following universal value a_M [3]

$$a_M = (1.2 \pm 0.2) \times 10^{-8} \text{ cm s}^{-2}, \quad a_M \approx \frac{1}{6}a_0 \approx \frac{1}{6}cH_0 \quad (3)$$

where a_0 is the observed cosmic acceleration. This discrepancy between Newtonian gravitation and the observed rotation curves can be explained by postulating that galaxies are surrounded by halos of dark matter. It seems difficult though to understand why the dark matter distribution becomes important precisely below the above mentioned critical acceleration (instead of beyond a critical distance from the galactic centre) and why this critical acceleration should be so close to the observed cosmic acceleration. There is also the possibility that a new fundamental force (let us call it the fifth force) becomes more significant than Newtonian gravitation, whenever the acceleration a falls much below the critical acceleration a_M . Keeping this in view, Milgrom proposed in 1983 that the acceleration a experienced by a test body of mass m in the presence of a source mass M is given by the following phenomenological law:

$$a = a_N = \frac{GM}{R^2} \quad \text{for } a \gg a_M, \quad a = \frac{\sqrt{GMa_M}}{R} \quad \text{for } a \ll a_M \quad (4)$$

In other words, the fifth force starts to dominate over Newtonian gravitation at sub-critical accelerations. This proposal is known as Modified Newtonian Dynamics (MOND) [3]. We do not interpret it as the breakdown of Newtonian gravitation/general relativity at low accelerations, but rather as the fifth force dominating Newtonian gravity. The test body of mass m universally experiences Newtonian gravity as well as the fifth force, due to the presence of the mass M . The acceleration due to both the forces is independent of the mass m of the test particle, but the fifth force is proportional to the square root of the source mass M and falls inversely with distance ($\sim \sqrt{M}/R$) as if it were the square-root of Newtonian gravitation ($\sim M/R^2$). Subsequently, we will view the MOND relation $a^2 = a_N a_M$ as a consequence of the introduction of the effective distance $R_{eff}^2 = RR_H$. This latter choice makes MOND analogous to Coulomb's law and paves the way for relativistic MOND as a $U(1)$ symmetry sourced by square root of mass.

An analogy could be made to the electroweak symmetry broken down to the weak force and electrodynamics. An electron in the presence of another electron experiences both the weak force and the much stronger Coulomb force. At energy scales approaching the electroweak scale ~ 1 Tev, the two forces have nearly equal strength and then get unified. At lower energies the electric force dominates the weak force but this does not mean the weak force law breaks down at low energies. It just means the weak force is comparatively weaker.

Analogously, the MOND force (i.e. the fifth force) dominates over Newtonian gravity (GR) at low accelerations, but that does not imply that GR is breaking down. In our work, MOND is to gravitation what electrodynamics is to the weak force. Electrodynamics (MOND) dominates over the weak force (GR) at low energies (accelerations). At high accelerations the fifth force and GR get unified (the darkelectro-grav symmetry $SU(2)_R \times U(1)_{YDEM}$).

The MOND phenomenology cannot derive the interpolating function between the Newtonian regime and the deep MOND regime: that can only come from the deeper theory from which MOND originates. Thus one introduces the unspecified interpolation function $\mu(x)$ relating the Newtonian acceleration a_N to the MOND acceleration g as

$$a_N = g\mu(g/a_M), \quad \mu(x) = 1 \text{ for } x \gg 1, \quad \mu(x) = x \text{ for } x \ll 1 \quad (5)$$

If one does not wish to introduce MOND as a fifth force, it can be presented as modified Poissonian gravity [25], by modifying the left hand side of the Poisson equation, with $a = -\nabla\phi$.

$$\nabla \cdot [\mu(|\nabla\phi|/a_0)\nabla\phi] = 4\pi G\rho \quad (6)$$

This modified Poisson equation can be derived from the following Lagrangian [26] :

$$\mathcal{L} = -\frac{1}{12\pi G a_0} ((\nabla\phi)^2)^{3/2} + \rho\phi \quad (7)$$

As Khoury notes: “However, as a theory of a fundamental scalar field, the non-analytic form of the kinetic term is somewhat unpalatable.” For the same reason one might be skeptical about modifying the left hand side of the Poisson equation; doing so will make it harder to relate MOND to other fundamental interactions, and to find a generalisation of GR which reduces to MOND in the non-relativistic limit, at low accelerations. We prefer to derive MOND from a Poisson equation in which the left hand side is intact as the conventional Laplacian, and the right hand side is a new source charge for a fifth force.

Nonetheless, as Milgrom writes [25], and we quote:

“Very interestingly, its deep-MOND limit,

$$\nabla \cdot [(|\nabla\phi|)\nabla\phi] = 4\pi G a_0 \rho \quad (8)$$

is invariant under space conformal transformations (Milgrom, 1997) [27]: Namely, beside its obvious invariance to translations and rotations, Eqn. (8) is invariant to dilatations, $\mathbf{r} \rightarrow \lambda\mathbf{r}$ for any constant $\lambda > 0$ [under which $\phi(\mathbf{r}) \rightarrow \phi(\mathbf{r}/\lambda)$], and to inversion about a sphere of any radius a , centered at any point \mathbf{r}_0 , namely, to

$$\mathbf{r} \rightarrow \mathbf{R} + \frac{a^2}{|\mathbf{r} - \mathbf{r}_0|^2} \quad (9)$$

with $\phi(\mathbf{r}) \rightarrow \hat{\phi}[\mathbf{r}(\mathbf{R})]$, and $\rho(\mathbf{r}) \rightarrow \hat{\rho}(\mathbf{R}) = J^{-1}\rho[\mathbf{r}(\mathbf{R})]$, where J is the Jacobian of the transformation (9). This ten-parameter conformal symmetry group of Eqn. (8) is known to be the same as the isometry (geometric symmetry) group of a 4-dimensional de Sitter space-time, with possible deep implications, perhaps pointing to another connection of MOND with cosmology (Milgrom, 2009a) [28].”

This important fact about MOND is very encouraging for us, because our proposed $U(1)_{DEM}$ symmetry is indeed the left-over unbroken symmetry from the deSitter like phase which precedes the darkelectro-grav symmetry breaking. This correspondence with deSitter provides justification for use of the effective distance $R_{eff}^2 = RR_H$, because doing so enables the aforesaid invariance under dilatations.

MOND can also be presented as a modification of the law of inertia, instead of modification of law of gravitation:

$$a = \frac{GM}{R^2} \text{ for } a \gg a_M; \quad \frac{a^2}{a_M} = \frac{GM}{R^2} \text{ for } a \ll a_M \quad (10)$$

In our proposal in this paper, MOND arises from a new (fifth) force obeying a modified law of inertia. Thus law of gravitation, as well as law of inertia, both get modified at low accelerations.

There have been several serious attempts to develop relativistic MOND, i.e. to generalise general relativity to a modified relativistic theory of gravitation, from which MOND will emerge in the non-relativistic approximation, for accelerations $a \ll a_M$. These include the TeVeS theory developed by Bekenstein [29], which includes a vector field and a scalar field besides the spacetime metric. TeVeS was originally claimed to be able to explain gravitational lensing and other cosmological observations, but is seriously constrained by observations in the solar system and in binary stars [25]. Another prominent relativistic

MOND has been proposed by Skordis and Zlosnik, dubbed RMOND, which is claimed to explain CMB anisotropies and the matter power spectrum [30].

Our reservation about these otherwise noteworthy relativistic generalisations is that they are expressly designed to meet the requirements of a relativistic MOND, and are not easy to motivate from first principles. The vector field and scalar field introduced in TeVeS are difficult to relate to the standard model of particle physics. The quantum field theoretic constraints on such theories are also challenging. On the other hand, the $U(1)_{DEM}$ proposed by us as RelMOND is a fallout of the $E_8 \times E_8$ unification, and was not invented to explain MOND. The use of \sqrt{m} comes from consideration of masses of quarks and leptons of the first fermion generation. Furthermore, the $SU(2)_R \times U(1)_{YDEM}$ gauge symmetry is likely to be a renormalizable quantum field theory.

There is an extensive literature and review on MOND and its extensive applications; we do not intend to review it here. The excellent SCHOLARPEDIA article by Milgrom is upto date and reviews MOND and its applications in all its aspects [25].

We make mention though of an ongoing related research of great importance: testing the law of gravitation in GAIA DR3 wide binaries [31–33]. A large number of such binaries are known in the solar neighbourhood of the Milky Way, and these have orbital radii ranging from about 200 AU to 30000 AU. The orbital acceleration crosses the critical MOND value a_M for radii around 1000 AU, transiting from the Newtonian regime (relatively low radii) to the alleged MOND regime (large radii). Around 2000 AU onwards, the measured acceleration should disagree with Newtonian prediction, if MOND is right. The analyses by Chae [34, 35] and by Hernandez [36, 37] shows that Newtonian gravitation is obeyed in the not so wide binaries, but breaks down for larger separations. Banik et al. disagree [38]. See however, Chae’s critical response to Banik, and the responses of Lasenby, Boyle, and especially of Hernandez, after the recent OSMU23 lecture by Banik [39]. See also the recent rebuttal by Hernandez and Chae [40]. To our understanding, the conclusion of Chae, and of Hernandez, that Newtonian gravitation breaks down below the critical acceleration a_M , is correct. It is remarkable that wide binaries have the same critical acceleration scale a_M as spiral galaxies do: there is no a priori reason for this to be so, unless the fifth force does indeed exist and begins to dominate gravitation below a_M . This anomaly in wide binaries cannot be explained by dark matter; therefore wide binaries are the likely smoking gun which will discriminate MOND from dark matter.

III. A BRIEF REVIEW OF VERLINDE'S ENTROPIC DERIVATION OF MOND

A. Introduction

Roughly 95% of our Universe consists of a nonbaryonic form of matter/energy exhibiting mysterious properties. This is sufficient reason to suspect that perhaps our knowledge of gravity is incomplete. General Relativity might not be universally applicable (*i.e.* not in all regimes of parameter space), and spacetime might not be an irreducible, primary concept. Instead, our *macroscopic* notions of spacetime and gravity might emerge from an underlying *microscopic* description.

Verlinde [12, 41] suggests that the observed dark energy and the phenomena usually attributed to dark matter have a common origin and can both be connected to *the emergent nature of spacetime* instead. The key idea is the competition between bulk degrees of freedom and surface degrees of freedom in a de Sitter Universe containing matter:

i) when surface degrees of freedom dominate the expression for the entanglement entropy (of spacetime plus matter), we have the standard GR regime;

ii) when bulk degrees of freedom (in the entanglement entropy) take over we enter the MOND regime.

This state of affairs corresponds to a *glassy* dynamics, *i.e.*, a mechanics for the microscopic qubits of information in which two time scales are at work:

i) a fast, short range dynamics that is responsible for the area law for the entanglement entropy;

ii) a slow, long distance dynamics that exhibits slow relaxation, aging and memory effects, and is responsible for the MOND regime. This is the dark-matter phase of emergent gravity.

The transition between the two occurs precisely as one crosses the cosmological horizon of de Sitter spacetime. Verlinde interprets this as a true *phase transition* in thermodynamic sense. The thermodynamic medium here would be a d -dimensional spacetime (de Sitter spacetime containing matter) exhibiting two phases:

i) the GR regime, corresponding to a *stiff* phase of this medium ;

ii) the MOND regime, corresponding to an *elastic* phase.

In this picture, dark matter is *not* to be understood as being made up of some kind of particles. Rather, due to this phase transition in the fabric of spacetime itself, gravitational

effects cease to be described by GR in order to exhibit MOND-like properties. GR regards spacetime as perfectly stiff; now we see that it can have elastic properties too. MOND is a consequence of the extremely small, but nonvanishing, elastic properties of spacetime. The net result is that *dark matter is an apparent phenomenon*, as its effects can be more economically understood in terms of the elastic properties of spacetime in this regime.

Altogether, Verlinde claims that:

- i)* spacetime emerges from the entanglement of qubits of information;
- ii)* their short-range entanglement (*i.e.*, between neighbouring bits) produces an entropy scaling as in the Bekenstein–Hawking area law;
- iii)* their long-range entanglement entropy (also called de Sitter entropy) gives rise to a volumetric law (contrary to an area law);
- iv)* de Sitter entropy is equipartitioned between all bits;
- v)* gravity is the force that describes the change in entanglement (*i.e.*, in spacetime) due to matter.

B. The flattening of galaxy rotation curves

The flattening of galaxy rotation curves occurs only when the gravitational acceleration GM/R^2 drops below a certain acceleration scale a_M , *i.e.*, whenever

$$\frac{GM}{R^2} < a_M \quad (11)$$

Here a_M is Milgrom’s acceleration scale [3, 42], related to the cosmic acceleration scale a_0 as per $a_M = a_0/6$, and

$$a_0 = cH_0 \simeq 10^{-10} \text{ ms}^{-2} \quad (12)$$

with H_0 the Hubble constant.

Denoting the observed gravitational acceleration by g_{obs} and the acceleration due to baryonic matter by g_{bar} , Milgrom’s proposal is that g_{obs} is a certain function f of g_{bar} such that

$$g_{\text{obs}} = f(g_{\text{bar}}) = \begin{cases} g_{\text{bar}} & \text{for } g_{\text{bar}} \gg a_M \\ \sqrt{g_{\text{bar}} a_M} & \text{for } g_{\text{bar}} \ll a_M \end{cases} \quad (13)$$

Eq. (13) above can be regarded as an equivalent restatement of the Tully–Fisher law.

Altogether we have two extreme regimes:

i) when $a \gg a_M$ we have the standard Newtonian regime;

ii) when $a \ll a_M$ we have the MOND regime, where Newton's second law gets modified.

In the intermediate regime $a \simeq a_M$, MOND makes no assumptions regarding the function f .

Verlinde's analysis [41] applies to de Sitter (dS) spacetime, because dS is the space that best fits our Universe according to current data. Now d -dimensional dS spacetime has the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \quad f(r) = 1 - \frac{r^2}{L^2}. \quad (14)$$

All computations are done under the assumption of spherical symmetry. At $r = L$ there is a cosmological horizon which carries a finite entropy and temperature. The surface acceleration κ is given in terms of the Hubble parameter H_0 and the Hubble scale L as

$$\kappa = cH_0 = \frac{c^2}{L} = a_0 \quad (15)$$

Then:

i) at scales much smaller than the Hubble radius L , gravity is well described by General Relativity (GR), because the entanglement entropy follows the Bekenstein–Hawking area law;

ii) at large distances GR breaks down and MOND sets in. This corresponds to the de Sitter entropy (which follows a volumetric law) taking over.

Equivalently:

i) gravity at accelerations greater than a_M obeys GR. Spacetime in this regime, although dynamical, is regarded as *stiff*, meaning *nonelastic*;

ii) this is opposed to gravity at accelerations below the scale a_M : in this MOND regime, gravity is modelled in ref. [12] as being due to the *elastic* properties of spacetime.

In GR, the definition of mass can be problematic. Strictly speaking, the ADM mass can only be defined at spatial infinity. However, dS spacetime has a cosmological horizon and no spatial infinity. In dS spacetime, an approximate analogue of the ADM mass can be defined

under some assumptions. It turns out to be given by

$$M = -\frac{1}{8\pi G} \int_{\mathcal{S}_\infty} \phi \partial_j n_j dA \quad (16)$$

where $dA = n_i dA_i$, n_i is the outward normal vector to the surface \mathcal{S}_∞ , and the latter is a large enough spherical surface enclosing the mass M placed around the origin. This surface \mathcal{S}_∞ must be far away from the origin, so the field produced by M can be approximately spherically symmetric; at the same time, \mathcal{S}_∞ may not be too close to the horizon.

C. Entropy as a criterion for a phase transition

1. Entropy increase when a bit traverses a horizon

The addition or subtraction of n bits (entering the horizon or leaving it) causes an increase or decrease ΔS in the entropy of the horizon. From Verlinde's first paper [41] we have the following result for the entropy increase of a horizon, as the latter is traversed by n bits of information:

$$\frac{\Delta S}{n} = -k_B \frac{\Delta\phi}{2c^2}. \quad (17)$$

Here k_B is Boltzmann's constant (hereafter $k_B = 1$), and $\Delta\phi$ is the difference in Newtonian potential between the states *before* the n bits traverse the horizon and *after* traversing it.

Thus the Newtonian potential ϕ keeps track of the depletion of horizon entropy per bit of information traversing it.

2. Entropy of empty dS space

De Sitter spacetime has a certain microscopic structure, the precise form of which is unknown (and fortunately also irrelevant for our purposes). In consequence we can assign dS spacetime an entropy. For the moment we regard dS spacetime as being *empty*, or devoid of matter. The expansion of empty dS spacetime is being driven by the dark energy. Verlinde computes the entropy of empty dS spacetime to be [12]

$$S_{DE}(r) = \frac{r}{L} \frac{A(r)}{4G\hbar}, \quad A(r) = \Omega_{d-2} r^{d-2} \quad (18)$$

The subindex DE stands for *dark energy*. The above expresses the entropy contained within a sphere of radius r and surface area $A(r)$. We draw attention to the *volume* dependence on the right-hand side of (18), because of the product $rA(r)$. Happily, when evaluated at $r = L$, Eq. (18) yields back the area-dependent Bekenstein–Hawking entropy. However, S_{DE} scales with the volume for $r < L$.

The entropy S_{DE} is carried by excitations of the qubits making up empty dS space that lift the negative groundstate energy to the positive value associated with the dark energy. In other words, dS entropy corresponds to the dark energy that drives the expansion of the Universe.

3. Entropy reduction of dS space due to the addition of matter

Our actual Universe is of course not empty. Applying the Bekenstein upper bound [43], Verlinde shows that the addition of a mass M causes the entropy of dS space to *decrease* by the amount

$$S_M(r) = -\frac{2\pi Mr}{\hbar} \quad (19)$$

because the horizon size is being reduced. The entanglement between the two sides of the horizon diminishes by the addition of this mass, hence the negative entropy.

4. The missing mass problem in entropic terms

We return to Eq. (11), which we would like to reexpress in entropic terms. Consider a spherical region with boundary area $A(r) = 4\pi r^2$ containing a total mass M . Then the gravitational phenomena attributed to dark matter occur only when the *area density* $\Sigma(r)$ of mass falls below a universal value determined by a_M :

$$\Sigma(r) = \frac{M}{A(r)} < \frac{a_M}{8\pi G} \quad (20)$$

We have replaced the condition (11), expressed in terms of accelerations, as condition (20), expressed in terms of surface density of mass. Next we recast the same condition in terms

of entropies. For this, we rewrite (20) more suggestively as

$$\frac{2\pi M}{\hbar a_M} < \frac{A(r)}{4G\hbar}, \quad (21)$$

We multiply through with r/L and use (15) to obtain

$$\frac{2\pi Mr}{\hbar} < \frac{r}{L} \frac{A(r)}{4G\hbar}. \quad (22)$$

Finally using (18) and (19) we have

$$|S_M(r)| < S_{DE}(r). \quad (23)$$

To summarise: the gravitational phenomena commonly attributed to dark matter occur whenever the inequality (23) holds. The bulk entropy S_{DE} scales with the volume, while the matter entropy S_M scales linearly with r . The observations on galaxy rotation curves tell us that the nature of gravity changes, depending on whether the matter added to dS space removes all or just a fraction of the entropy S_{DE} of dS space.

Therefore we have two regimes:

i) the regime when $S_M(r) < S_{DE}(r)$, which corresponds to low surface mass density $\Sigma(r)$ and low gravitational acceleration: this is the MOND, or sub-Newtonian, or *dark matter* regime;

ii) the regime when $S_M(r) > S_{DE}(r)$, which describes Newtonian gravity.

Verlinde's goal is to explain why the laws of emergent gravity differ from those of General Relativity (GR) precisely when the inequality (20) (equivalently (23)) holds. His conclusions are:

i) at scales much smaller than the Hubble radius, gravity is well described by GR because the entanglement entropy is still dominated by the area law of the vacuum; this is identified as the *stiff* phase of spacetime;

ii) at larger distances and/or longer time scales the bulk dS entropy leads to modifications of the above laws. Precisely when the surface mass density falls below the value (20), the reaction force due to the thermal contribution takes over from the usual gravity governed by the area law. This is identified as the *elastic* phase of spacetime, in which MOND gravity takes over.

5. Newtonian gravity in terms of surface densities

Motivated by the previous arguments, next we will rewrite the familiar laws of Newtonian gravity in terms of a *surface mass density* vector Σ .

Given a Newtonian potential ϕ and the corresponding acceleration $g_i = -\partial_i\phi$ we define

$$\Sigma_i = \frac{d-2}{d-3} \frac{g_i}{8\pi G} \quad (24)$$

This is the usual gravitational acceleration vector g_i , with some convenient normalisation. The latter is so chosen that the differential expression of Gauss' law in d -dimensional dS spacetime now reads

$$\nabla \cdot \Sigma = \rho \quad (25)$$

That Σ qualifies as a surface mass density follows from the equivalent integral expression of the Gauss law

$$\int_{\mathcal{S}} \Sigma \cdot d\mathbf{A} = M \quad (26)$$

where M is the total mass enclosed by the surface \mathcal{S} . Finally the gravitational self-energy U_{grav} of a mass distribution can also be expressed in terms of Σ :

$$U_{\text{grav}} = \frac{1}{2} \int dV g_i \Sigma_i \quad (27)$$

This rewriting of Newtonian gravity in terms of surface densities will facilitate its interpretation in terms of elasticity theory.

D. The elastic phase of emergent gravity

1. Elastic moduli in terms of gravitational parameters

Verlinde next proves that the ADM-like definition of mass (16) can be naturally translated into an expression for the strain tensor. Given the Newtonian potential ϕ , the corresponding elastic displacement field u_i is postulated to be

$$u_i = \frac{\phi}{a_0} n_i \quad (28)$$

where n_i is the outward unit normal to a surface \mathcal{S}_∞ . The latter encloses a mass given by

$$M = \frac{a_0}{8\pi G} \int_{\mathcal{S}_\infty} (n_j \varepsilon_{ij} - n_i \varepsilon_{jj}) dA_i \quad (29)$$

where ε_{ij} is the strain tensor for the displacement field u_i . Multiplying both sides of (29) by the acceleration scale a_0 we obtain a force:

$$Ma_0 = \int_{\mathcal{S}_\infty} \sigma_{ij} n_j dA_i \quad (30)$$

where we have identified the stress tensor of the dark-matter elastic medium to be

$$\sigma_{ij} = \frac{a_0^2}{8\pi G} (\varepsilon_{ij} - \varepsilon_{kk} \delta_{ij}) \quad (31)$$

This yields *the elastic moduli of the dark-matter medium*:

$$\mu = \frac{a_0^2}{16\pi G}, \quad \lambda = -\frac{a_0^2}{8\pi G} \quad (32)$$

2. A derivation of the Tully–Fisher relation

Dark matter causes a gravitational pull, an acceleration g_D which scales with $\sqrt{M_D}$, the square root of the dark mass M_D . This is opposed to baryonic matter, whose acceleration g_B scales with the baryonic mass M_B . Verlinde finds that in d -dimensional dS spacetime one has the following analogue of (13):

$$g_D^2 = g_B a_M, \quad a_M = \frac{d-3}{(d-2)(d-1)} a_0 \quad (33)$$

When $d = 4$, Eq. (33) is equivalent to the Tully–Fisher relation (13). Then $a_M = a_0/6$, which is the acceleration scale appearing in Milgrom’s phenomenological fitting formula (13).

Eq. (33) is *a theoretical* derivation of the *phenomenological* Tully–Fisher law Eq. (13). This derivation from first principles can be seen as one of the main achievements of Verlinde’s paper.

3. Apparent dark matter in terms of baryonic matter

Using the previous dictionary between elastic and gravitational quantities, Verlinde derives an expression for the density of apparent dark matter as a function of baryonic matter. Namely, Σ_D as a function of the *baryonic* Newtonian potential ϕ_B :

$$\left(\frac{8\pi G}{a_0}\Sigma_D\right)^2 = \left(\frac{d-2}{d-1}\right)\frac{1}{a_0}\partial_i(\phi_B n_i) \quad (34)$$

In the spherically symmetric case, and when $d = 4$, the above can be integrated within a sphere of radius R to yield

$$\int_0^R \frac{GM_D^2(r)}{r^2} dr = \frac{1}{6}M_B(R)a_0R \quad (35)$$

where

$$M(R) = \int_0^R \rho(r)A(r) dr \quad (36)$$

is the total mass inside the radius R . Eqs. (34) and (35) describe *the amount of apparent dark matter in terms of the amount of baryonic matter*; as such they allow to make direct comparison with observations. In ref. [44] it is claimed that the agreement is good.

IV. MOND AS GRAVITY IN 2+1 DIMENSIONS

Any Riemannian manifold \mathbb{M} has an associated Laplacian operator ∇^2 . The latter has a Green function $G(p, p')$ satisfying the Poisson equation $\nabla_p^2 G(p, p') = -\delta(p - p')/\sqrt{\det g_p}$. Thus $G(p, p')$ can be regarded as the Newtonian potential created at point $p \in \mathbb{M}$ by a unit mass located at point $p' \in \mathbb{M}$. Now $G(p, p')$ becomes singular as $p \rightarrow p'$. When \mathbb{M} is 2-dimensional, on general grounds we expect this singularity to be proportional to the logarithm of $d(p, p')$, the geodesic distance between the two points. Here we prove this conjecture by explicitly computing the Laplacian Green functions for the 2-dimensional sphere and for the 2-dimensional hyperbolic space.

A. Green's functions for the Laplacian in two dimensions

B. The plane \mathbb{R}^2

Given the Euclidean metric on the plane

$$ds^2 = dr^2 + r^2 d\varphi^2, \quad (37)$$

where $0 < r < \infty$, $0 < \varphi < 2\pi$, the corresponding Laplacian operator reads

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \quad (38)$$

The Green function $G(r, \varphi; r', \varphi')$ for a massless scalar on \mathbb{R}^2 satisfies the equation

$$\nabla^2 G(r, \varphi; r', \varphi') = \frac{-1}{r} \delta(r - r') \delta(\varphi - \varphi') \quad (39)$$

Without loss of generality we can assume $r' = 0$; this point is the origin. Moreover, by rotational symmetry, the Green function cannot depend on φ . We will thus denote the Green function more simply by $G(r)$ and look for the solution to

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) G(r) = \frac{-1}{r} \delta(r) \quad (40)$$

The solution reads

$$G(r) = [A - \Theta(r)] \ln r + B \quad (41)$$

where $\Theta(r)$ is the Heaviside step function. We will set $A = 0 = B$ and consider

$$G(r) = -\Theta(r) \ln r \quad (42)$$

which, for $r > 0$, simplifies to

$$G(r) = -\ln r \quad (43)$$

Since r equals the geodesic distance between the origin and the point (r, φ) , our statement follows.

C. The sphere \mathbb{S}^2

Given the standard round metric on the unit sphere \mathbb{S}^2 ,

$$ds^2 = d\theta^2 + \sin^2 \theta d\varphi^2, \quad (44)$$

where $0 < \theta < \pi$, $0 < \varphi < 2\pi$, the corresponding Laplacian operator reads

$$\nabla^2 = \frac{\partial^2}{\partial \theta^2} + \cot \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \quad (45)$$

The Green function $G(\theta, \varphi; \theta', \varphi')$ for a massless scalar on \mathbb{S}^2 satisfies the equation

$$\nabla^2 G(\theta, \varphi; \theta', \varphi') = \frac{-1}{\sin \theta} \delta(\theta - \theta') \delta(\varphi - \varphi') \quad (46)$$

Without loss of generality we can assume $\theta' = 0$; this point is the north pole. Moreover, by rotational symmetry, the Green function cannot depend on φ . We will thus denote the Green function more simply by $G(\theta)$ and look for the solution to

$$\left(\frac{d^2}{d\theta^2} + \cot \theta \frac{d}{d\theta} \right) G(\theta) = \frac{-1}{\sin \theta} \delta(\theta) \quad (47)$$

The change of variables $x = \cos \theta$ reduces (47) to

$$(1 - x^2)g''(x) - 2xg'(x) = \delta(x) \quad (48)$$

where we have set $g(x) = G(\theta)$. Eq. (48) is solved by

$$g(x) = \frac{1}{2} \ln \left(\frac{1-x}{1+x} \right) [A - \Theta(x)] + B \quad (49)$$

where A, B arbitrary integration constants. We may set $A = B = 0$ to obtain

$$G(\theta) = \frac{1}{2} \ln \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right) \Theta(\cos \theta) \quad (50)$$

as Green's function for the Laplacian on the (upper hemisphere of the) 2-sphere, *i.e.*, when $0 < \theta < \pi/2$; an analogous expression can be written for the lower hemisphere.

The point $\theta = 0$ is a singularity of $G(\theta)$: for $\theta \rightarrow 0^+$ we find

$$G(\theta) = -\ln \theta + \dots \quad (51)$$

where the dots stand for regular terms in θ . Now, in a small neighbourhood of the point considered, the tangent plane can be identified with the sphere. To this order of accuracy, the Green function (51) exhibits the expected logarithmic singularity in the geodesic distance.

D. Hyperbolic space \mathbb{H}^2

The metric on hyperbolic space \mathbb{H}^2 is

$$ds^2 = d\varrho^2 + \sinh^2 \varrho d\varphi^2 \quad (52)$$

where $0 < \varrho < \infty$, $0 < \varphi < 2\pi$. The corresponding Laplacian operator reads

$$\nabla^2 = \frac{\partial^2}{\partial \varrho^2} + \coth \varrho \frac{\partial}{\partial \varrho} + \frac{1}{\sinh^2 \varrho} \frac{\partial^2}{\partial \varphi^2} \quad (53)$$

The Green function $G(\varrho, \varphi; \varrho', \varphi')$ for a massless scalar on \mathbb{H}^2 satisfies the equation

$$\nabla^2 G(\varrho, \varphi; \varrho', \varphi') = \frac{-1}{\sinh \varrho} \delta(\varrho - \varrho') \delta(\varphi - \varphi') \quad (54)$$

For the same reasons as above it suffices to solve the ordinary differential equation

$$\left(\frac{d^2}{d\varrho^2} + \coth \varrho \frac{d}{d\varrho} \right) G(\varrho) = \frac{-1}{\sinh \varrho} \delta(\varrho) \quad (55)$$

We apply the change of variables $x = \cosh \varrho$ to obtain

$$(x^2 - 1)g''(x) + 2xg'(x) = -\delta(x), \quad (56)$$

where we have set $g(x) = G(\varrho)$. Eq. (56) coincides with (48), hence an analogue of (50) applies:

$$G(\varrho) = \frac{1}{2} \ln \left(\frac{1 + \cosh \varrho}{1 - \cosh \varrho} \right) \quad (57)$$

Again Taylor–expanding around $\varrho = 0$ and dropping irrelevant constants we arrive at the expected logarithmic singularity in the geodesic distance:

$$G(\varrho) \simeq -\ln \varrho + \dots \quad (58)$$

V. DARK ELECTROMAGNETISM AS THE ORIGIN OF RELATIVISTIC MOND

We demonstrate that MOND can be written as Coulomb’s law analogous to Maxwell’s electrodynamics, by using an effective distance. Energy conservation along with a modified inertia law can then be used to show that, written as Coulomb’s law, MOND mimics cold dark matter, including on cosmological scales. Furthermore, in the deep MOND regime, this formulation is the non-relativistic limit of the $U(1)_{DEM}$ gauge theory.

We have in the deep MOND regime that the acceleration a of a test particle in the field of a mass M is given by

$$a = \sqrt{Ga_M} \frac{\sqrt{M}}{R} = \sqrt{L_P c^2 a_M} \frac{\sqrt{M/m_{Pl}}}{R} = c^2 \sqrt{\frac{L_P}{L_M}} \frac{\sqrt{M/m_{Pl}}}{R} \quad (59)$$

where a_M is Milgrom’s acceleration constant; $L_M = c^2/a_M$ is the MOND radius.

We will assume that the MOND force F on the test particle of mass m is to be obtained by multiplying the acceleration by $\sqrt{mm_{Pl}}$. We write the force in terms of dimensionless masses, so as to try to make it look more and more like electrodynamics:

$$F = \sqrt{mm_{Pl}} a = m_{Pl} c^2 \sqrt{\frac{L_P}{L_M}} \frac{\sqrt{M/m_{Pl}} \sqrt{m/m_{Pl}}}{R} \quad (60)$$

Assuming that we live in a deSitter universe, we multiply and divide by the Hubble radius $R_H = cH_0^{-1}$ which is also the deSitter horizon, and we introduce the effective distance R_{eff} given by $R_{eff}^2 \equiv RR_H$.

$$F = \sqrt{mm_{Pl}} a = m_{Pl} c^2 R_H \sqrt{\frac{L_P}{L_M}} \frac{\sqrt{M/m_{Pl}} \sqrt{m/m_{Pl}}}{R_{eff}^2} \quad (61)$$

Now this looks like Coulomb’s law, in terms of the effective distance R_{eff} . If a spatial point \mathbf{x} is at a distance $|\mathbf{x}|$ from the observer, it has to be stretched by a factor R_H . We can discuss the covariance of this procedure, but in a Robertson-Walker universe with cosmic

time, this procedure seems well-defined.

We assume that the Milgrom constant a_M is η times the cosmic acceleration $a_0 = cH_0$ and also that $a_0 = \beta a_{Pl}(L_P/R_H)$ is the scaling down of the Planck acceleration due to the deSitter expansion. Thus, $L_M = c^2/a_M = c^2/\eta a_0 = c^2 R_H/\eta \beta a_{Pl} L_P$. We can hence write the force as

$$F = A \frac{\sqrt{M/m_{Pl}} \sqrt{m/m_{Pl}}}{R_{eff}^2} \quad (62)$$

where

$$A = m_{Pl} c \sqrt{R_H} \sqrt{L_P^2 a_{Pl} \eta \beta} = \hbar c \sqrt{\eta \beta R_H / L_P} \longrightarrow A = \frac{3}{2} \hbar c \sqrt{\eta \beta R_H / L_P} \quad (63)$$

The factor of $3/2$ is deliberately introduced so as to get consistency with Verlinde's result and consistently derive the famous factor of $1/6$ relating Milgrom's constant to the cosmic acceleration. We will take (62) as the defining force law of the $U(1)_{DEM}$ interaction, with A as defined in (63), with the factor of $3/2$ included. MOND is to be derived from this force law, even though initially we started from MOND so as to motivate this Coulomb like force law.

Below we consider generalising this to a fully relativistic theory for the square-root mass current. The theory can be expected to be derivable from an action principle, just like Maxwell electrodynamics. For now, let us continue with the spherically symmetric Coulomb case.

This force law has an interesting parallel with, and an important difference from, Maxwell's electrodynamics. We can write Coulomb's law as $F = \hbar c (e^2/\hbar c)/R^2$. The charge is expressed in dimensionless units here, so a multiplication by $\hbar c$ appears, just as for the above gravity case. However the gravitational coupling is scaled by a factor dependent on epoch, via the Hubble radius (with the understanding that $R_H = c^2/a_0$ and epoch dependence, if any, would come from the in-principle-allowed time variation of the cosmic acceleration). And gravity uses the effective distance, which is like a scaling of the actual distance.

The force law can be derived from a potential ϕ via $F = d\phi/dR_{eff}$ so that

$$\phi = -A \frac{\sqrt{M/m_{Pl}} \sqrt{m/m_{Pl}}}{R_{eff}} \quad (64)$$

We would now like to write down the energy conservation equation in the deep MOND

regime, given this potential, and from that equation derive Verlinde's central equation (7.40) in his paper [12]. The energy conservation equation is obtained by starting from the equation of motion for the test mass m at R having velocity $v = dR/dt$.

$$\sqrt{m}\sqrt{m_{Pl}}\dot{v} = -\frac{d\phi}{dR_{eff}} = -\frac{dR}{dR_{eff}} \frac{d\phi}{dR} \quad (65)$$

The left hand side of this equation is a modified inertia law, and in fact is such that the MOND acceleration is independent of the square-root mass of the test particle. Thus we still have the equivalence principle, but this time arising from cancellation of square-root mass when the dark charge is identified with the inertial square root mass.

Multiplying both sides by v and noting that $dR_{eff}/dR = R_H/2R_{eff}$ we can write

$$\sqrt{m_{Pl}}\sqrt{m} \frac{1}{2} \frac{d}{dt}[v^2] + \frac{2R_{eff}}{R_H} \frac{d}{dt}\phi_{eff} = 0 \quad (66)$$

If we make the crucial assumption that the time-dependence of $2R_{eff}/R_H$ can be ignored this equation can be integrated to get the following expression for a conserved energy, after substituting the form of the potential:

$$-\frac{2E}{\sqrt{m}\sqrt{m_{Pl}}} \frac{1}{R^2} + \frac{\dot{R}^2}{R^2} = \frac{6\sqrt{Ga_0\eta}\sqrt{M}}{R^2} \quad (67)$$

As is done in the Newtonian derivation of the Friedmann equation (converting force law into energy conservation) we equate the right hand side term to the source term of the Einstein equations, as if sourced by an apparent dark matter distribution $M_D(R)$ (Verlinde's notation).

$$\frac{6\sqrt{Ga_0c\eta}\sqrt{M}}{R^2} = 8\pi G\rho_D = 8\pi G \frac{M_D}{4\pi R^3/3} = \frac{6GM_D}{R^3} \quad (68)$$

assuming a constant density and a uniform apparent dark matter distribution. Squaring both sides gives

$$\frac{GM_D^2}{R} = \eta a_0 MR \quad (69)$$

which is consistent with Verlinde's eqn. (7.40) in [12] if we assume $\eta = 1/6$. From here, following Verlinde, MOND law can be easily derived.

It seems interesting that we get the same result for apparent dark matter as Verlinde

does. This can be considered a support for the proposed $U(1)_{DEM}$ symmetry. Furthermore the introduction of the effective distance can be interpreted as a stretching of the distance R to the larger distance R_{eff} and reminds us of an elastic medium. We should explore how to relate our effective distance to Verlinde's elasticity approach to MOND: the two might be related to each other. Note that the amount of apparent dark matter M_D is proportional to the square-root of the actual matter M . We hope to derive these results from first principles in future work.

We can also try to now prove that the total amount of apparent dark matter is about five times ordinary matter. Verlinde's equation (7.40) is

$$\int_0^R \frac{GM_D(r')^2}{R'^2} = \frac{1}{6}a_0M(R)R \quad (70)$$

Assuming a uniform density ρ_D one can integrate the left hand side, after expressing mass in terms of density, to get

$$\frac{GM_D^2}{5R} = \frac{1}{6}a_0M_D(R)R \longrightarrow M_D = \sqrt{\frac{5}{6G}}\sqrt{a_0MR} = \sqrt{\frac{5}{3}}M \quad (71)$$

The last equality follows by considering the entire universe, and writing the mass M in terms of density, assuming critical density $\rho = 3H_0^2/8\pi G$ which gives $H_0 = 1/2GM$. For R we have assumed the value Hubble radius $R_H = H_0^{-1}$ which is also the deSitter horizon.

This is the contribution to apparent dark matter from the Coulomb part of the potential energy. If we assume that each of the three vector components also contribute in equal measure, we get that the total apparent dark matter is $4 \times \sqrt{5/3} = 5.16$ times ordinary matter. This agrees well with the standard LCDM model according to which dark matter to ordinary matter ratio is about 5.3.

A very important point is that only particles with non-zero rest mass take part in dark electromagnetism, just as only particles with non-zero electric charge take part in electromagnetism. Hence there is no $U(1)_{DEM}$ interaction between photons and baryonic matter: from this point of view the apparent dark matter derived above is a perfect mimicker of dark matter. It will produce an additional gravitation-like attraction but it will not have any impact on the CMB anisotropy produced by baryons interacting with electromagnetic radiation on the last scattering surface. We can as usual study the growth of linear density

perturbations by working with apparent density fluctuations in apparent dark matter.

Furthermore, the potential energy of the dark electromagnetic field serves as a source on the right hand side of Einstein equations, just as cold dark matter does. Therefore, in so far as causing gravitational lensing is concerned, the DEM field mimics cold dark matter.

The non-relativistic limit of dark electromagnetism (dark equivalent of Coulomb's law) proposed above is the limit of relativistic dark electromagnetism, patterned after Maxwell electrodynamics.

We propose the following action principle for dark electromagnetism and general relativity. It is patterned entirely after the action for Maxwell's electrodynamics coupled to sources, in a curved spacetime. The electromagnetic field is replaced by the DEM field.

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x \sqrt{-g} \mathcal{L}_{matter} - \frac{1}{16c} \int d^4x \sqrt{-g} F_{ij} F^{ij} + A \int d^4x \sqrt{-g} D_i J^i \quad (72)$$

The last term couples the dark electromagnetic potential D_i to the current density J^i of square-root mass, obtained by multiplying the latter by four velocity. The coupling constant A was defined earlier in Eqn. (63), wherein R_H is to be understood as $R_H = c^2/a_0$. The source for gravity is the energy-momentum tensor of mass and the energy momentum tensor of the dark field. The dark current is given by

$$J^i = \sum_a \frac{\sqrt{m/m_{pl}} c}{\sqrt{-g}} \delta(\mathbf{y} - \mathbf{y}_a) \frac{dx^i}{dx^0} \quad (73)$$

Here, the spatial distance \mathbf{y} is the effective distance, i.e. $|\mathbf{y}|^2 = R_H |\mathbf{x}|$ and the time $t = x^0/c$ is the cosmic time used in Robertson-Walker metric and Friedmann equations. The dark potential is also a function of the effective spatial distance \mathbf{y} and not of \mathbf{x} . Thus, if we define $y^i = (t, \mathbf{x})$, then $F_{ij} = \partial_{y_i} D_j(y) - \partial_{y_j} D_i(y)$. The interaction term of the dark current does not contribute to the energy-momentum tensor which appears on the right hand side of Einstein equations, because the $\sqrt{-g}$ in the denominator of the expression for current density cancels the $\sqrt{-g}$ in the numerator in the expression for interaction action (last term in action above). This is the same as in Maxwell's electrodynamics, but in the present case of dark electromagnetism it has profound significance. Namely, the source term for GR (it being the energy-momentum tensor proportional to mass m) is completely distinct from the source term for dark electromagnetism, this being the current density of square-root

mass. Two masses m_1 and m_2 interact both via GR and via DEM, and one interaction dominates over the other, depending on the magnitude of the acceleration. Furthermore, the introduction of the effective distance in DEM and the specific use of cosmic time as time, breaks Lorentz invariance. DEM as relativistic MOND picks up a specific reference frame, which we take to be the rest frame of the CMB.

The second last term is the action for the DEM field, made from its field tensor, which also couples to gravitation. Its energy-momentum tensor will contribute as a source in the Einstein field equations. Varying the action wrt the metric gives Einstein's field equations sourced by dust and DEM field; varying wrt DEM field gives Maxwell-like equations coupling DEM field to the current density of square-root mass, and varying wrt particle position gives geodesic equation of motion, which now also includes the effect of the DEM field as an external non-gravitational force.

More explicitly, variation of the action will give: Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{ik} = \frac{8\pi G}{c^4} (T_{\mu\nu}(\text{matter}) + T_{\mu\nu}(\text{DEM})) \quad (74)$$

Maxwell-like equations sourced by current density of square-root mass: all written as functions of the effective spatial distance, and cosmic time:

$$F_{:k}^{ik} = -\frac{4\pi}{c} A j^i \quad (75)$$

Geodesic equation, which has also an external force included (so that motion becomes non-geodesic), with the Maxwell-like force being proportional to square-root mass (analogous to electric charge), and a function of the effective spatial distance

$$mc \frac{Du^i}{ds} = mc \left(\frac{du^i}{ds} + \Gamma_{kl}^i u^k u^l \right) = \frac{\sqrt{m/m_{Pl}}}{c} A F^{ik} u_k \quad (76)$$

As and when the effects of DEM are insignificant, Lorentz invariance and GR are recovered, as expected. These field equations will reduce (in the Newtonian approximation, and in the homogeneous isotropic cosmological approximation) to the analysis in Section V above.

The treatment of the exact field equations is left for future work, and if the analysis

can be done, it might even yield the sought for interpolating function mediating Newtonian gravitation and MOND.

Milgrom [28] writes that: “... one may conjecture that the MOND-cosmology connection is such that local gravitational physics would take exactly the deep-MOND form in an exact de Sitter universe. This is based on the equality of the symmetry groups of dS^4 and of the MOND limit of the Bekenstein-Milgrom formulation [45] both groups being $SO(4,1)$. The fact that today we see locally a departure from the exact MOND-limit physics—i.e., that the interpolating functions have the form they have, and that a_0 is finite and serves as a transition acceleration—stems from the departure of our actual space-time from exact dS^4 geometry: The broken symmetry of our space-time is thus echoed in the broken symmetry of local physics.” Our proposal that $U(1)_{DEM}$ is the remnant broken symmetry after breaking of $SU(2)_R \times U(1)_{YDEM}$ is entirely in support of this conjecture of Milgrom.

VI. DERIVATION OF VERLINDE’S ENTROPIC CRITERION, FROM DARK ELECTROMAGNETISM

Consider the epoch of left-right symmetry breaking where also the $SU(2)_R \times U(1)_{YDEM}$ symmetry is broken. The deSitter expansion (as in the octonionic theory) ends with the formation of compact objects. The $U(1)$ symmetry remains unbroken, like in the electro-weak sector, and becomes the $U(1)_{DEM}$ symmetry which we are currently examining. $U(1)_{DEM}$, like MOND, is a scale invariant theory and carries memory of the deSitter phase. GR arises as a result of symmetry breaking. Consider a black hole arising from spontaneous localisation, which in fact is how the deSitter expansion ends. As Verlinde shows [12], the formation of a localised compact object reduces the deSitter entropy. The criterion for MOND to be dominant is that this reduction in entropy (which is area proportional) is less than the volume entropy of deSitter in the volume occupied by the compact object. This is equivalent to saying that memory of deSitter is retained under these conditions, and that $U(1)_{DEM}$ dominates over GR.

We can try to derive Verlinde’s entropy criterion by starting from our $U(1)_{DEM}$ theory. Let us start by asking what is the temperature of a black hole whose radius is such that its surface gravity is less than the critical MOND acceleration? Assuming that the radius R of the black hole is given as in GR and hence $R = 2GM$, the acceleration on the surface is,

assuming that the effective radius of the black hole is $R_{eff} = \sqrt{RR_H} = \sqrt{2GMR_H}$

$$a = \sqrt{GMa_M}/R \longrightarrow a = \sqrt{GMa_M}/R_{eff} = \sqrt{GMa_M}/\sqrt{2GMR_H} \sim a_0 \quad (77)$$

where we have neglected the numerical coefficient for now. The interesting point is that this acceleration is independent of the mass of the black hole, and if we associate a temperature with the black hole, it being proportional to the surface gravity, the temperature is a_0 , just as for the deSitter horizon, and independent of the mass of the black hole. This is an example of deSitter memory being retained in the $U(1)_{DEM}$ dominated deep MOND regime.

The entropy of the black hole is given by

$$dQ = TdS \rightarrow dS = dQ/T = dM/a_0 \rightarrow S \sim M/a_0 \quad (78)$$

which is consistent with Verlinde's result. In the deep MOND regime, this entropy is less than the deSitter volume entropy, directly as a consequence of our $U(1)_{DEM}$ theory.

VII. COUPLING CONSTANTS IN THE DARKELECTRO-GRAV THEORY

For the electroweak sector $SU(2)_L \times U(1)_Y$, the derived fundamental constants are the low energy fine structure constant $\alpha_{fsc} \equiv e^2/\hbar c$ and the weak mixing angle (Weinberg angle) θ_W , this latter being the solution of the trigonometric Eqn. (56) of our paper [4]. The fine structure constant is made from the parameters α and L appearing in the Lagrangian of the theory, as displayed e.g. in Eqn. (6) of the just mentioned paper. The constants of the electroweak sector are expressible in terms of the fine structure constant and the weak mixing angle, along with the value of the Higgs mass m_H whose value is to be predicted from cosmological downscaling (caused by the deSitter-like inflationary expansion) from the original Planck scale value of the Higgs mass. It is significant that the standard model Higgs comes from the right sector in the left-right symmetric model (whereas the standard model forces arise from the left sector). The second Higgs, associated with the left sector, is a newly predicted Higgs which is electrically charged.

Thus the weak isospin g (i.e. the $SU(2)_L$ coupling) is given by $g = e/\sin \theta_W$ and the weak hypercharge g' (the $U(1)_Y$ coupling) is given by $g' = e/\cos \theta_W$. The Higgs mass is estimated as follows. When the mass ratios are computed in the octonionic theory, we assume that the

lightest of the masses, i.e. the electron mass, is one in Planck units. (Likewise, the charge of the down quark, it being the smallest electric charge, is set to one while determining the fine structure constant). Hence, the Higgs mass is initially about $3 \times 10^5 m_P \sim 10^{24}$ GeV because the Higgs is a composite of standard model fermions, and is expected to obtain maximum contribution from the top quark, which at about 173 GeV is about 3×10^5 heavier than the electron. An inflation by a factor of 10^{61} scales this mass down by a factor $10^{61/3}$, to the value of about 10^3 GeV. This sets the weak coupling Fermi constant $G_F^0 \sim 1/v^2$ (where $v \sim 246$ GeV is the Higgs VEV), to about 10^{-6} GeV $^{-2}$ whereas the experimentally measured value for the Fermi constant is about 10^{-5} GeV $^{-2}$.

This derivation of the reduced coupling constant $G_F^0 = G_F/(\hbar c^3) \sim g^2/M_W^2 c^4$ helps us arrive at a reasonable estimate of the W boson mass from first principles. We also observe that the Fermi constant has dimensions of length squared (same as G_N) and can be written as $G_F^0 \sim g^2(m_P/m_W)^2 m_P^{-2} \sim g^2(m_P/m_W)^2 G_N/(\hbar c)$. The scaling down of the W mass from its Planck scale value is responsible for the weak force becoming so much stronger than gravitation. In this theory, G_N remains unchanged with epoch.

Knowing m_W , the mass of the Z boson is determined, as is conventional, by the relation $m_Z = m_W/\cos \theta_W$. This way we have a handle on the fundamental constants and parameters of the electroweak sector (Higgs mass, Fermi constant, fine structure constant, weak mixing angle, masses of weak bosons, weak isospin, and hypercharge). For understanding why there is sixty-one orders of magnitude of inflation, which ends at the electroweak scale, please see [20] - the same result is also supported by the idea that the electroweak symmetry is broken below a critical acceleration (see Discussion section below).

Let us now discuss the coupling constants and parameters of the right-handed dark electrograv (DEM) sector, $SU(2)_R \times U(1)_{YDEM}$ staying as close as possible to the above discussion for electroweak sector. The DEM symmetry is broken along with the electroweak symmetry. It can also be shown using the electric charge operator, i.e. the number operator which is associated with the $U(1)_{em}$ symmetry, that W^+ and W^- have electric charge +1 and -1 respectively and that Z^0 is electrically neutral. The corresponding situation for the $SU(2)_R \times U(1)_g$ symmetry is interesting, because here the $U(1)_{DEM}$ number operator defines square root of mass (in Planck mass units); it does not define electric charge. Consequently, W_R^+ and W_R^- have square root mass +1 and -1 respectively, and hence their range of interaction is limited to Planck length. They will also have an extremely tiny electric charge,

some seventeen orders of magnitude smaller than the charge of the electron (analogous to the W mass being so small on the Planck scale). Whereas the $U(1)_{YDEM}$ boson (and the dark photon it transforms to) will have zero mass and zero electric charge. Z_R^0 will be massless, and will have an extremely tiny electric charge (like the W_R bosons). It is possible that emergent gravitation is mediated at the quantum level by the Z_R^0 and the dark photon. They take the place of the spin 2 graviton, in this theory.

The place of the fine structure constant is taken by the mass of the electron. The Weinberg angle satisfies the same equation and hence has the same value as in the electroweak case. Thus the right sector analogs of the couplings g and g' can be obtained. GR is the result of the breaking of the $SU(2)_R$ symmetry [i.e. the quantum-to-classical transition]. The remaining unbroken symmetry is dark electromagnetism $U(1)_{DEM}$ which is the proposed origin of relativistic MOND. The cosmological origin of MOND is briefly discussed in [20].

During the deSitter like inflationary phase, $E_8 \times E_8$ symmetry is operational, and includes as a subset the unbroken electroweak symmetry $SU(2)_L \times U(1)_Y$ as well as the darkelectrograv symmetry $SU(2)_R \times U(1)_{YDEM}$. Below the critical acceleration these symmetries are broken, giving rise to the emergence of classical spacetime (precipitated by the localisation of fermions). Near compact objects the gravitationally induced acceleration (GR/Newton) is higher than the critical acceleration and GR dominates. In the far zone, the acceleration is below the critical acceleration: this is the deep MOND regime where the unbroken symmetry $U(1)_{DEM}$ dark electromagnetism dominates. This zone is the buffer between deSitter horizon and the GR zone, and it has been identified also in Verlinde's work using his entropy considerations.

All (left-handed) particles take part in the weak force, and all electrically charged particles take part in electromagnetism. Analogously, all right-handed particles take part in the $SU(2)_R$ interaction, whereas all particles with non-zero square-root mass take part in dark electromagnetism.

VIII. DISCUSSION

We somehow tend to think that R is the genuine distance and that the effective distance R_{eff} is introduced by brute force. This need not be true, and the actual situation can be the other way round. Let us rename the effective distance R_{eff} as true distance R_{true} . We do that

for the following reason. In our approach the universe begins out as a deSitter-like universe, and the formation of structures such as black holes (GR dominated near BHs, MOND farther out) ends the deSitter phase. Let R_{true} be the physical distance of some point, with respect to the observer. We propose that as a result of the spontaneous localisation which causes a classical structure such as a black hole to form, the distance R_{true} shrinks to R in the same ratio that the Hubble radius (event horizon distance) bears to R_{true} . Therefore,

$$\frac{R}{R_{true}} = \frac{R_{true}}{R_H} \quad (79)$$

This provides some physical basis, in terms of initial conditions, for using the effective distance.

A. Critical acceleration

It has been demonstrated by earlier researchers that if an inertial observer observes a spontaneously broken symmetry, then a Rindler observer concludes that the symmetry is not broken, provided the acceleration is above a certain critical value. See e.g. [46] and [47]. Padmanabhan was one of the early researchers to show this result: Indeed Section 7 and in particular Eqn. (7.15) of the work of Padmanabhan [48].

The 2017 paper [49] shows the critical acceleration for the electroweak case: This result appears significant for what we are doing with dark electromagnetism arising from the breaking of $SU(2)_R \times U(1)_{YDEM}$, in the early universe. It helps understand that classicality and GR emerge as a result of the acceleration of the universe coming down below the critical value. This critical value happens to be the same as the current acceleration of the universe.

These results could have important implications in early universe cosmology. In particular, it could be that the electroweak symmetry breaks when the acceleration of a quasi-deSitter expanding universe falls below a critical value (assuming the inflation-like phase ends at the electroweak scale).

In our research we are investigating if Milgrom's MOND arises as the result of breaking of an $SU(2)_R \times U(1)_{YDEM}$ symmetry, which is the right-handed counterpart of electroweak. After spontaneous symmetry breaking the $U(1)$ becomes $U(1)_{DEM}$, which is dark electromagnetism. We are looking into whether this fifth force is an alternative to dark matter, and

the sought for theoretical basis of MOND. The critical acceleration result could be relevant in establishing the SSB criterion.

B. Limiting values

Consider the quantity: $\frac{\sqrt{M}}{R_{eff}^3}$ which can be expanded around a spatial point as $\frac{\sqrt{\rho_0 R^3}}{R^{3/2} R_H^{3/2}}$ which has the finite limit $\sqrt{\rho_0/R_H^3}$. This reinforces the use of the effective distance. In a non-spherical situation the effective distance between two spatial points (having coordinate difference $\mathbf{x} - \mathbf{x}'$) is defined by new coordinates $\mathbf{y} - \mathbf{y}'$ such that $|\mathbf{y} - \mathbf{y}'|_{eff}^2 = R_H |\mathbf{x} - \mathbf{x}'|^2$

C. Advantages in considering dark electromagnetism

We summarise here some of the key advantages of $U(1)_{DEM}$ symmetry:

1. It arises from first principles from $E_8 \times E_8$ theory.
2. It is a relativistic gauge theory.
3. It is plausible that the unbroken $SU(2)_R \times U(1)_{YDEM}$ symmetry is renormalizable, and is the correct theory of quantum gravity.
4. $U(1)_{DEM}$ is sourced by square root of mass, just as desired by MOND.
5. Only particles with non-zero rest mass take part in $U(1)_{dem}$. A photon does not interact with matter through $U(1)_{DEM}$. Therefore the additional force created by the baryon - $U(1)_{DEM}$ interaction is the perfect dark matter mimicker. It will explain CMB anisotropies for the same reason that dark matter explains CMB anisotropies. It will also mimic dark matter vis a vis gravitational lensing.
6. There is a natural connection with deSitter because $U(1)_{DEM}$ is the leftover unbroken symmetry from deSitter.
7. We are able to derive Verlinde's results for apparent dark matter and entropy criterion for MOND
8. Earlier researchers have demonstrated that the electroweak symmetry is broken below a certain critical acceleration, and restored above it. The same result can be expected to hold for its right-handed counterpart, this being our GR-DEM theory. Analogous to electro-weak, we could call it darkelectro-grav.

The dark photon - the gauge boson that mediates quantised DEM, could be thought of

as dark matter. Its detection in the laboratory may however be beyond current technology. Same could be said about dark electromagnetic waves, though they could well be the early dark radiation [50, 51] that has been proposed as one possible solution to the Hubble tension. Even though the dark photon can be called the sought for dark matter, what is noteworthy is that the associated DEM field is MONDian in character, and we have a newly predicted fifth force mimicking dark matter, but not a new fermionic elementary particle as dark matter. From the point of view of fundamental physics, this difference (i.e. is dark matter fermionic or bosonic) is significant. It decides whether there are only four fundamental forces, or more than four.

Acknowledgements: We thank Sukratu Barve, Kinjalk Lochan, and Cenalo Vaz for helpful discussions. We thank the Vielberth Foundation for support. C.P. thanks the Academic Research Sabbatical Program of the University of Regensburg for enabling this cooperation. JMI was partially supported by FEDER/MCIN under grant PID2022-142407NB-I00 (Spain).

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